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Securities settlement fails network and buy-in strategies

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Abstract

In the context of securities settlement, a trade is said to fail if on the settlement date either the seller does not deliver the securities or the buyer does not deliver funds. Settlement fails may have consequences for the parties directly involved and for the system as a whole. Chains of fails, for example, could lead to gridlock situations and large volume of fails can affect the liquidity and smooth functioning of financial markets. In this paper, we consider UK government bonds (gilts) and UK equities settlement data to examine the determinants of settlement fails and to explore the network characteristics of chains of settlement fails with the aim of identifying an optimal strategy to conduct a buy-in process that could resolve cascades of fails.

Key words: Settlement, post-trade process, settlement discipline, cascade effects, buy-in.

JEL classification: E42, G23.

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1 Introduction

In the context of securities settlement, a trade is said to fail if by the intended settlement date either the seller does not deliver the securities or the buyer does not deliver funds. In reality, at least in the UK, fails usually occur because the seller did not deliver the security, while fails due to lack of cash can occur but are very rare. In general, settlement fails are not treated as a default event by market participants but more as an operational friction. This recognizes the fact that, while cash is fungible, securities are less so as counterparties may require a specific security type (that is, a specific ISIN[1]) and some securities may not be easily available in the market for purchase or borrowing.

Fails can have a duration of more than one day and can be a consequence of any one of a number of factors, including operational issues (e.g. lack of communication between front and back offices), the impact of chains of fails ("daisy chains"), or imbalances between supply and demand (e.g. demand may increase when investors choose to sell a security without owning it and need to borrow it for delivery). In particular, the balance between supply and demand will have an impact on the incentives to settle on time. For example, failing to deliver securities may result in a benefit to the seller if the cost of borrowing a security is higher than the cost of failing, in which case short sellers may intentionally choose not to deliver, an example of what Fleming and Garbade (2002) called a "strategic fail". In a low interest rate environment, the incentive for the seller to settle is weakened, while a buyer will have limited re-investment options and would therefore have a greater interest in settling the trade, as illustrated in Figure 1.

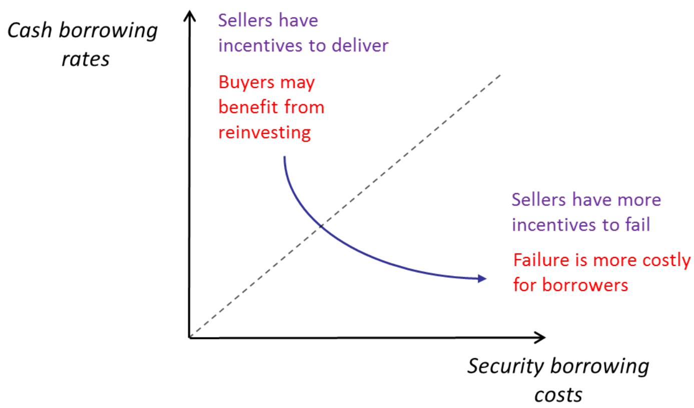


Figure 1: Illustration of the trade-off between cash and security borrowing costs and its potential impact on the incentives to fail on a settlement obligation. Here we only consider the two parties involved in the transaction and do not include any potential systemic effects.

Fails affect the two parties involved because, during the fail period, they remain exposed to replacement cost risk and they may have to assume an unexpected liquidity risk exposure; but fails can also have consequences for the system as a whole, as timely settlement allows market participants to make contingent plans, contributing to the depth

[1]The International Identification Security Number (ISIN) is a code used to uniquely identify individual securities.

and liquidity of the financial markets and to their smooth functioning.

Moreover, it is often the case that participants rely on receiving a security which they have to deliver. In such cases, an initial idiosyncratic fail can propagate through the system in a cascade of fails. A typical example is that of a firm B that, as part of a short-selling strategy, borrows a security from dealer A. To keep a matched book, the dealer A then borrows the security from another participant C. If, at the time of giving back the security, B fails to give it back to the dealer A, then the dealer will fail to return the security to C (producing what is called a "daisy chain" of fails). If such activity is widespread (as it seems to be), fails can easily propagate and deteriorate liquidity in the market, becoming a systemic issue. In their analysis of the US markets, for example, Iyer and Macchiavelli (2017) show that chains of fails are indeed systemic, with the dealer-specific pass-through from fails-to-receive to fails-to-deliver being around 90% to 100% across collateral types and time periods.

Cascades of fails can lead to gridlock situations where no participant is able to deliver the security to their counterparty. And large volumes of fails could also affect lending, because lenders may decide to withhold securities if they believe that they might not receive them back promptly at the end of the agreements.[2] From the perspective of market regulators, an increase in fails can indicate impairment in market function or an early warning that a market participant is experiencing liquidity or operational problems.

Given the potential negative externalities resulting from a failed settlement, the industry and regulators have sought to increase settlement efficiency through the implementation of settlement discipline frameworks, which may include 1) measures to prevent fails; for example, by providing services to facilitate security borrowing and lending, 2) measures to discourage fails, like increasing the cost of failing through penalty schemes,[3] and 3) measures to mitigate the impact of fails and avoid the propagation of liquidity risk; for example, by establishing special procedures for handling fails that remain unresolved after a certain period of time (ECB, 2011).[4]

Among the possible measures to mitigate the impact of fails, one of the most commonly used is the "buy-in" process, which allows the buyer to receive the securities from a third party when a fail persists over a given period of time, with the participants settling any price differences between the original transaction and the buy-in price. In some cases, activating a buy-in process is an option that can be exercised by the fail-to-receive party,

[2]This was observed, for example, in the US Treasury market after the operational disruption and large volume of fails produced by the 11/9 events. The withdrawal of some institutional members from the specialist market was widely noted (Fleming and Garbade, 2002).

[3]In the US, for example, the Treasury Market Practices Group (TMPG) introduced in May 2009 a fails charge for U.S. Treasury and agency debt securities. This mechanism applies a penalty charge of 300 bps less the prevailing Fed Funds rate, reflecting the natural economic cost of failing. Thus, as market rates move higher, the penalty charge reduces, and with Fed Funds at 3%, the charge becomes zero. This ensures that the cost of failing is always 300 bps or higher, even in a low-rate environment. The charge is paid directly by the failing party to the failed-to party. In the years after the introduction of the TMPG fails charge, settlement fails decreased considerably (Fleming et al., 2014a). In 2010, the Japanese Government debt market introduced a similar penalty regime (Bank of Japan, 2011).

[4]We should also note that the Capital Requirements Regulation (CRR) requires that, with the exception of repos, transactions which remain unsettled 5 days after their due delivery dates (i.e. T+5) need to be capitalised (CRR, III, Title V, Article 378).

while in other cases the process may be initiated automatically after a certain period of time has passed.[5] On the other hand, because of the potential complexity of a network of fails, designing an efficient buy-in strategy raises a number of challenges.[6] For example, when a network of fails has resulted in closed chains of fails it may be difficult to determine which participants need to be bought in and by which amounts in a way that the outcome is consistent across all the nodes in the network and the curing effect of the buy-in is maximized.

The academic literature on settlement fails has mostly focused on the US markets (Evans et al., 2009; Fleming and Garbade, 2002, 2005) and on exploring the determinants of settlement fails. More recent studies have also investigated the EU markets (Corradin et al., 2017) and have considered the systemic nature of settlement fails (e.g. Iyer and Macchiavelli (2017), including modelling the impact of the default of a major settlement participant (Devriese and Mitchell, 2005). However, to the best of our knowledge there are no studies on the network characteristics of settlement fails or on determining the efficiency of buy-in regimes. Moreover, because of the differences in regulation and market practices, we should bear in mind that studies may not be easily comparable across borders, as there is potential for a wide variation of behaviour and structures between jurisdictions.

In this paper, we contribute to the literature first, by examining the determinants of settlement fails in UK government bonds (gilts) and in large-cap UK equities; second, by analyzing the network structures of cascades of settlement fails and developing an algorithm to identify fails due to cascade effects; and third, by characterizing - from a theoretical perspective - the minimum set of nodes that need to be bought in so that a network of fails can be resolved.

For the first part of the analysis, our choice of markets is driven by two considerations: first, the FTSE 100 and gilt markets are systemically important and therefore relevant from a financial stability perspective. Second, these are very liquid markets, which allows us to assume that the fails we observe are not mainly driven by low levels of supply. Given the rarity of cash fails in these markets, in this paper we focus only on securities fails.

The rest if the paper is organized as follows: In the next section we review the literature on settlement fails. In Section 3 we describe the institutional background and the main characteristics of the UK gilt and equity markets. Section 4 presents the data used in the analysis together with summary statistics. In Section 5 we present some stylized facts about the concentration and persistence of fails across securities and participants. Section 6 examines the network characteristics of fails, both from static and dynamic perspectives, and propose an algorithm to identify fails due to cascade effects. In Section 7 we discuss,

[5] For example, in the Spanish markets, as part of the settlement process, the Central Securities Depository (CSD) automatically initiates a buy-in if the fail has not been solved after 5 days (see `https://www.clearstream.com/clearstream-en/products-and-services/market-coverage/europe-t2s/spain/settlement-process-spain-1280964`). In the EU, the Central Securities Depositories Regulation (CSDR, 2014) that was published in August 2014, includes a settlement discipline regime which incorporates, among other measures to improve settlement efficiency, the requirement of mandatory buy-in after a fail has persisted over a period of time. This period is dependent on the asset type and on the liquidity of the financial instrument.

[6] For a reference on the overall risk associated to different layers of complexity in a network of interactions, see Battiston, S. et al. (2016).

in a general setting, a buy-in strategy that cures a network of settlement fails at a minimal cost. Section 8 concludes with a short summary of the results.

2 Related literature

In broad terms, we can identify two strands in the literature on settlement fails: one aims at identifying and understanding the determinants of settlement fails while the other focuses on assessing their systemic nature. While our analysis sits within this second trend, we will begin this section by briefly recalling some of the results around the determinants of settlement fails.

As we have mentioned, previous literature has mostly focused on the U.S. equity and Treasuries markets. In the case of the U.S. Treasuries, for example, Fleming and Garbade (2002) found evidence of strategic fails; in particular, they found that while the number of settlement fails initially rose as a consequence of the destruction of records and communication facilities during the 11 September attacks, it remained high even after those facilities were restored because the method typically used to avert or remedy a fail - borrowing a security through a special repurchase agreement - proved as costly as failing to deliver the security. On the other hand, Evans et al. (2009) provided evidence of strategic behaviour in the U.S. equity markets: looking at the stocks in the Russell 300 index, they show that in most situations where it is hard to borrow, market makers may choose to fail instead of borrowing the stock. Moreover, they found that the alternative to fail is valuable for the pricing and trading of options.

Fleming and Garbade (2005) analyze data on U.S. Treasury and other securities. Their results reveal substantial variation in the frequency of fails over the 1990-2004 period and suggest that surges in fails sometimes result from operational disruptions, but often reflect insufficient incentives for market participants to avoid failing. Boni (2006) shows that, in the case of U.S. stocks, the likelihood of persistent fails decreases with institutional ownership and increases for low “glamour” stocks (that is, those stocks with low book-to-market ratios), which supports the hypothesis that equity and option market makers strategically failed to deliver shares that were expensive to borrow. They also document that many of the firms that are themselves in a fail-to-receive position and do not exercise their option of a forced delivery (through a buy-in) are themselves responsible for fails-to-deliver in other stocks, and they provide empirical evidence consistent with the hypothesis that many firms avoid exercising their buy-in option simply because they are unwilling to earn a reputation for forcing delivery and hope to receive a quid pro quo in their own strategic fail.

Analyzing a spike in fails in the U.S. Treasury markets during June 2014, Fleming et al. (2014b) observe that sequential fails of several benchmark securities accounted for majority of fails, but that fails in seasoned securities (defined as securities issued more than 180 days prior) were also elevated. They provide evidence that traders may decide to strategically fail when the premium to get a specific security becomes high, even if they incur in penalties. In a later post, Fleming and Keane (2016) explore the reasons for the upward trend in fails in seasoned securities that has been observed in recent years. They establish a set of stylized facts about the duration and security characteristics of these fails and conclude that the increase can be explained by more issues failing and in larger quantities, but not by longer fails episodes. Moreover, the fails are increasingly dispersed across securities, with little apparent concentration in particular tenors or vintages of securities.

The link between naked short selling and fails to delivery has also been a recurrent topic. Using a sample of NYSE stocks over a 42-month period, Fotak et al. (2014) investigate the impact of equity fails on market quality. Their results show that the ability to fail has a beneficial impact on liquidity and on the pricing efficiency of equity markets, and that any fails that arise from short sales should contribute to price discovery and liquidity on the day of trade in the same way as timely delivered short sales.

In a recent paper, Corradin et al. (2017) observed that specialness - the premium of procuring a specific security in the repo market - increased for bonds bought by the Eurosystem and could be linked to the level of fails: a 1bp change on specialness increases the probability of fail by 0.37%. They argue that short-selling traders had to pay a net specialness premium to close their positions and therefore may have decided to fail on their delivery. Indeed, bonds that were bought under the program were more likely to underly a failed transaction.

From the perspective of understanding the systemic nature of settlement fails, Devriese and Mitchell (2005) use a simulation model to analyze the potential impact on a securities settlement system of a disruption caused by the default of a large participant. More recently, a study by Iyer and Macchiavelli (2017) of fails across various asset classes in the U.S. (Treasuries, Agency MBS, Agency Notes, and Corporate Securities) during the 2007-09 period showed that, at the dealer level, failing to receive a security is passed almost one-to-one into failing to deliver the same security, which suggests a high degree of collateral re-hypothecation together with the inability or unwillingness to borrow or buy the needed securities. They also show that the implementation in 2009 of the fails charge for Treasury securities marginally improved the situation.

Our paper contributes to this strand of the literature by examining the network of settlement fails in UK government bonds (gilts) and in large-cap UK equities, and by quantifying the prevalence of cascades of settlement fails in these markets. Furthermore, from a more general perspective, we develop an algorithm to identify fails due to cascade effects and we characterize the minimum set of nodes that need to be bought in so that a network of fails can be resolved.

3 Institutional framework and market structure

Our study will focus on the UK government bonds (gilts) and equities markets. Both gilts and UK equities are settled in CREST (the UK's Securities Settlement System), which is operated by Euroclear UK & Ireland (EUI), the UK's Central Security Depository (CSD). For sterling and euro transactions, CREST offers real time delivery versus payment (DvP) in central bank money. Currently, CREST has a daily settlement value in excess of £1 trillion across all the instruments it settles. It runs several settlement cycles during the day and transactions that fail to settle in one cycle are automatically positioned for the next settlement cycle. This means that daily reports of fails reflect trades that have not been able to settle throughout the whole day, including fails outstanding from previous days.

EUI offers direct membership to corporate entities (including brokers, international banks, custodians and investment houses). These direct participants may provide access for other retail investors (including private individuals and companies).

3.1 Gilts cash and repo markets

Gilts are nominal fixed-coupon bonds issued by Her Majesty's Treasury (HMT) on behalf of the UK government and settle one day after the trade has taken place ("T+1"). Although they are listed on the London Stock Exchange (LSE), the vast majority of trading in gilts takes place over the counter, with trades being negotiated bilaterally over the phone or through electronic trading platforms. At the time of writing, there were 19 dealers that participated directly in the gilt auctions (they are commonly known as Gilt-Edged Market Makers or 'GEMMs').[7] In the secondary-market, GEMMs generally trade through interdealer brokers (IDBs) which act as intermediaries for anonymous trading between the market-makers. The main investors in gilts include UK-based insurance companies and pension funds, and overseas investors such as governments, central banks, sovereign wealth funds and asset managers.

Gilts also trade in the repo market, both in the form of general collateral repo (when general gilts are used as collateral) or specific repo (when specific gilts are used as collateral). In fact, the vast majority of sterling-denominated repo involves the sale and repurchase of gilts. Like the secondary gilt market, the gilt repo market is intermediated by a group of dealer banks, and there is significant overlap between the communities of GEMMs and repo dealers. Almost all interdealer gilt repo trading is at overnight term, and the vast majority of interdealer gilt repo trades are centrally cleared through LCH.Clearnet Ltd (Bicu et al., 2017).

3.2 FTSE 100

In the case of equities, we will focus on those included in the FTSE 100 index, which comprises the 100 UK stocks with highest market capitalization. In contrast with gilts, equities are traded mostly at exchanges (in which case, they settle at $T+2$), a fact which

[7]These were Barclays, BNP Paribas, Citigroup, Deutsche Bank, Goldman Sachs, HSBC, Jefferies, JP Morgan, Lloyds, Merrill Lynch, Morgan Stanley, Nomura, Royal Bank of Canada, Royal Bank of Scotland, Santander, Scotiabank, Toronto-Dominion Bank, UBS and Winterflood Securities. The list of current GEMMs can be found at https://www.dmo.gov.uk/responsibilities/gilt-market/market-participants/

may explain some of the differences we find in the structure of fails in these markets. For example, if we accept the view that the presence of high frequency trading (HFT) benefits liquidity, then the participation of HFT in exchange-traded equity markets may have a positive impact on settlement efficiency.

4 Data and summary statistics

Our main source of data includes reports from EUI on daily settlements in CREST of gilts and equities. The sample covers a 6-month period, from 3 October 2016 to 31 March 2017, representing 127 business days. The transactional information includes the International Identification Security Number (ISIN), the identities of the direct member acting as buyer and of the direct member acting as seller, the type of transaction, the quantity (volume) and value of the transaction, the intended settlement date (ISD) and the status of the transaction at the end of the day. In the case of gilts, volumes are reported with reference to the bonds' percentage price.

Since one member can hold multiple accounts at EUI, we consolidate the accounts according to the parent institution and we exclude from the network analysis transactions between accounts belonging to the same institution. In addition, we focus only on Delivery versus Payment (DvP) transactions.[8]

In the dataset, the settlement fails are recorded at the end of each business day until finally being settled. Therefore, by comparing the intended settlement date (ISD) with the current fail date we know how many days a transaction has being outstanding.

Since data is at settlement level, we do not know how trades were initially originated. In particular, we cannot distinguish whether a trade arises from an outright sale or as part of a repo trade. Moreover, we only observe positions by the direct CSD participant and cannot identify which trades are originated by its clients.

Table 1 presents summary statistics of the data. Fails are accounted on a daily basis and include fails that have been outstanding up to 5 days. To measure the intensity of fails we estimate the daily fail ratio (FR_t), defined for each day t as the volume of failed settlement transactions reported on day t scaled by the total volume of transactions on that day. On average, there are more daily transactions in FTSE 100 than in gilts; however, the volume per transaction is lower, resulting in lower daily volumes in FTSE 100. On the other hand, the ratio of fails FT_t is higher in FTSE 100 than in gilts (around 1.7 times) and almost twice as volatile.

To see how fails are distributed and evolve through time, Figure 2 displays the distribution of the daily volume of transactions that failed to be settled per day for all ISINs and all sellers together with the evolution of the fail ratio during the sample period. Although there are large variations in the daily fail ratio FR_t through the period, with perhaps the

[8] Our analysis does not include other types of operations for which 'fail' may have a different interpretation, as in the case of stocks withdrawals (used for the removal of electronic stock from CREST to be recertificated in paper form), stock deposits (dematerialisation of paper stock into electronic form within CREST), or transfers of stock between member accounts. In the case of gilts, we have also excluded from the analysis self-collateralization repo transactions (these are automated repos between settlement bank clients an their settlement banks).

Table 1: Summary statistics of equity and gilts fails in the period from 3 October 2016 to 31 March 2017 (127 business days). Fails are accounted on a daily basis and include fails that have been outstanding up to 5 days. FR_t is the daily fail ratio and S.D. is the standard deviation.

	Gilts				FTSE 100			
	Min	Max	Mean	S.D.	Min	Max	Mean	S.D.
Number of transactions ('000s)	6.03	14.2	10.8		21.3	33.4	28.6	
Successful								
Value (£bn)	92.6	261.05	186.03	27.79	6.16	37.9	13.2	4.83
Volume (bn)	70.54	189.65	140.64	21.14	1.46	8.84	4.01	1.08
Failed								
Value (£bn)	2.26	18.74	7.58	2.71	0.83	4.19	1.73	0.53
Volume (bn)	1.82	15.38	5.86	2.17	0.15	0.70	0.29	0.098
FR_t	0.019	0.095	0.040	0.013	0.026	0.165	0.069	0.023

exception of a drop in the volume of gilt trades during the December holiday period, we cannot distinguish any particular trend.

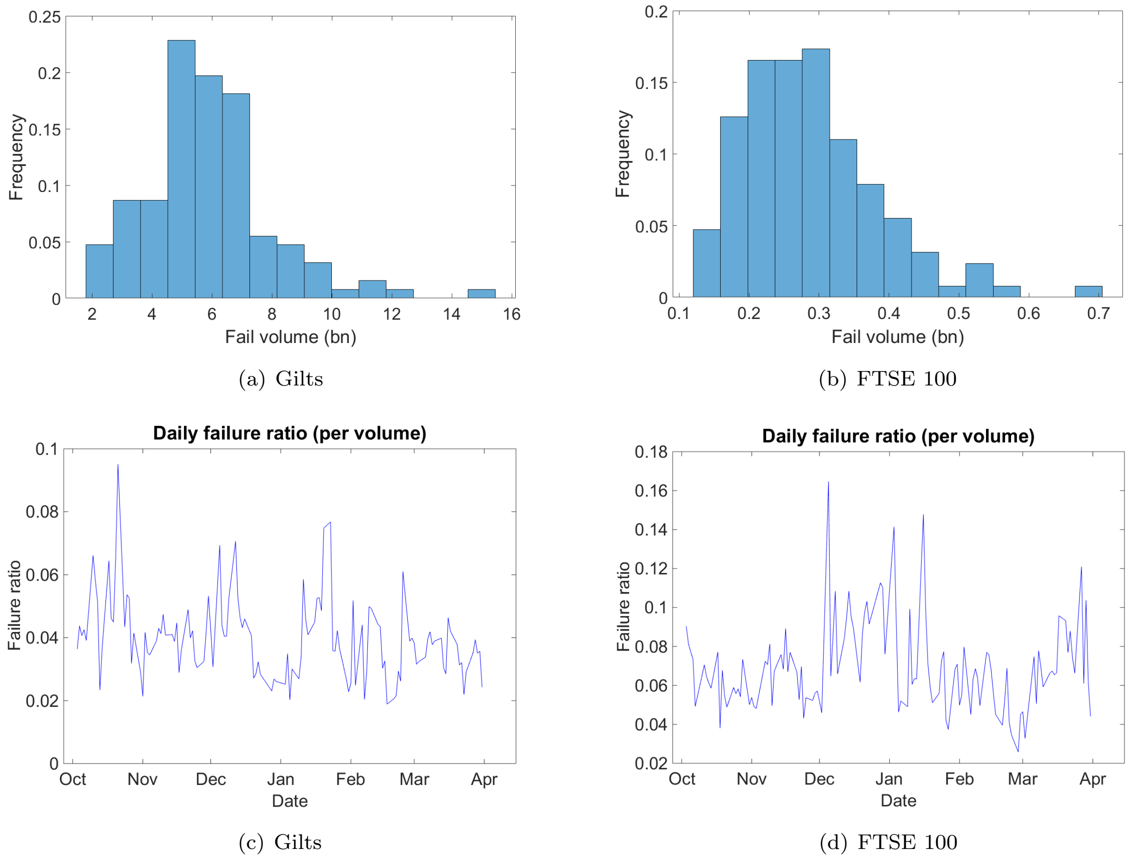
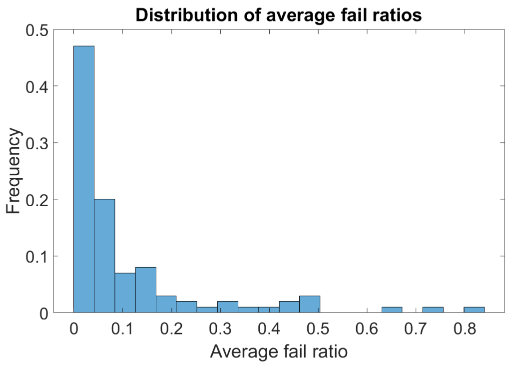
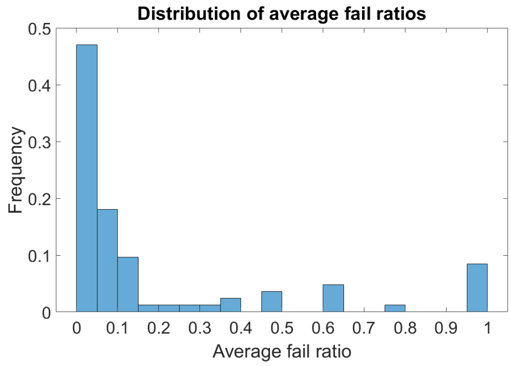


Figure 2: These figures are based on the daily volume of gilts and FTSE 100 trades that failed to settle during the sample period from October 2016 to March 2017. The top charts show the relative frequency distribution of daily volumes of fails. The bottom charts illustrate the daily fail ratio (FR_t).

Another aspect to consider is how fails are distributed across participants. For example, Figure 3 shows the distribution of participants according to their fail ratio. While in both markets almost half of the participants have fail ratios close to zero, some participants, especially in FTSE 100, have fail ratios close to 1 (at the right tail of the distribution). As we will see in Section 5, these participants tend to have very small volume of trades.



(a) Gilts



(b) FTSE 100

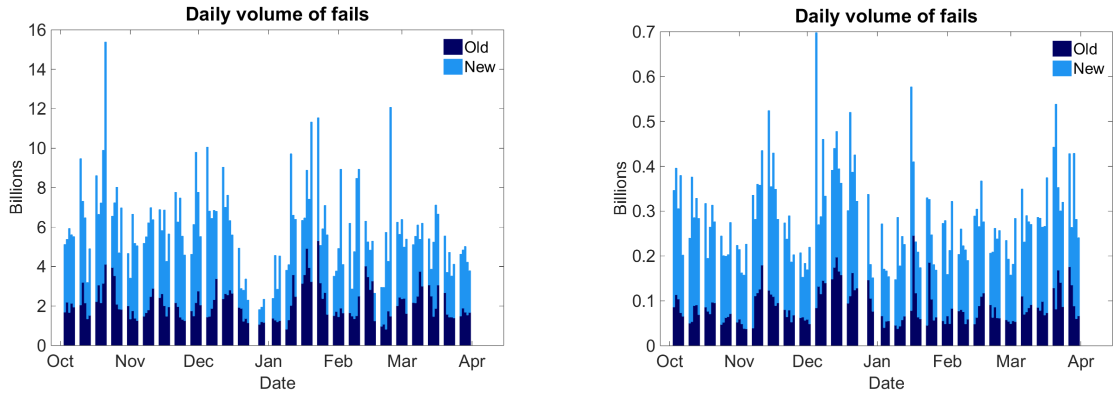
Figure 3: These figures show the relative frequency distribution of sellers according to their average daily fail ratio in the gilts and FTSE 100 markets during the period between October 2016 and March 2017.

4.1 The persistence of fails

We finish this section by documenting how long it takes for failed transactions to settle. For any given transaction k, let $t_{ISD}(k)$ be its intended settlement day and let $t_S(k)$ be the day when the transaction was finally settled (or canceled). In particular, the fact that a transaction has failed means that $t_S(k) > t_{ISD}(k)$. For each t, let $\mathcal{F}_t$ be the set of transactions that failed on day t. In other words, $k \in \mathcal{F}_t$ if and only if $t_S(k) > t \geq t_{ISD}(k)$. For each transaction $k \in \mathcal{F}_t$, we define the *number of days outstanding* of k at time t as the difference $n_t(k) = t - t_{ISD}(k)$. In particular, on each day t, we can distinguish between *new* ($n_t(k) = 0$) and *old* ($n_t(k) > 0$) fails. To illustrate the persistence of fails through time, Figure 4 shows the daily volume of fails during the sample period, distinguishing between old and new fails. On average, old fails account for around 38% of the total daily value of fails in gilts, while for FTSE 100 they do so for 31%.

Given the above definitions of old and new fails, we consider the persistence of fails across individual securities and across sellers. Panels (a) and (b) in Figure 5 show the average daily volume of fails across ISINs according to the business days between the intended settlement day and the day in which the trade was finally settled (or canceled). In our sample, if securities with 5 of more days of fails were to be bought in, then the average daily volumes of buy-ins would amount to around 0.023 bn for gilts and 0.005 bn for FTSE 100. In value terms, these would represent £0.032 bn and £0.016 bn, respectively.

Panels (c) and (d) of Figure 5 show the distribution of sellers according to the average length of time their failed transactions take to be resolved. This average is obtained by averaging across ISINs and across all days in the sample period. The results show that gilt market participants tend to fail for longer periods. In particular, 20% of failing gilt



(a) Gilts (b) FTSE 100

Figure 4: These figures show the daily volume of failed settlements in the gilts and FTSE 100 markets during the sample period from October 2016 to March 2017, distinguishing between new and old fails. For gilts, the average daily volume of new fails is 3.74bn and the average ratio of old fails with respect to the total number fails is 38%. For FTSE 100, the average volume of new fails is 0.2 bn and the average ratio of old v.s. total fails is 31%. The gaps between bars in the plots correspond to non-business days (weekends and bank holidays).

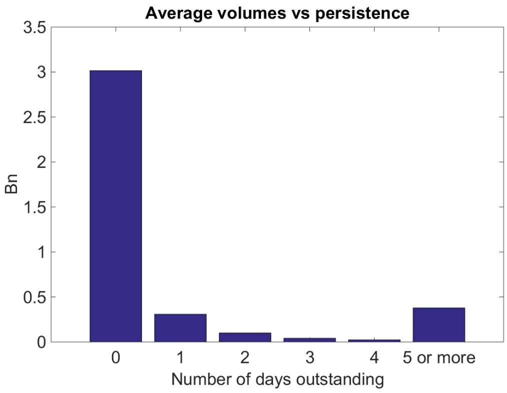
sellers failed for 5 business days or more, compared to less than 10% in the FTSE 100 market.

5 The determinants of settlement fails

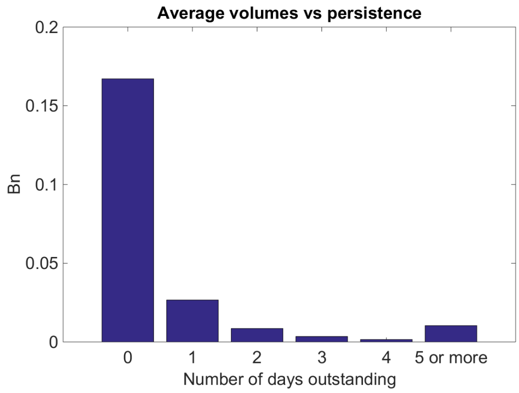
In general, it is common to assume that settlement fails typically occur for one (or more) of the following reasons:

- Operational problems: Communications between the seller's front and back office fails and the back office is unable to provide the securities to settle trades struck by the front office. Technological disruptions can also impede settlement.
- Liquidity problems: The security is not available, for example because of
  - Cascades of fails: a market participant was relying on receiving the same securities it was supposed to onward deliver.
  - Supply/demand imbalances: For example, short sellers may be unable to cover the position ahead of the intended settlement date, because of high demand of a particular security.
- Strategic behaviour: The seller may deliberately fail if borrowing costs are high or if it can profit from using the security in another trade.

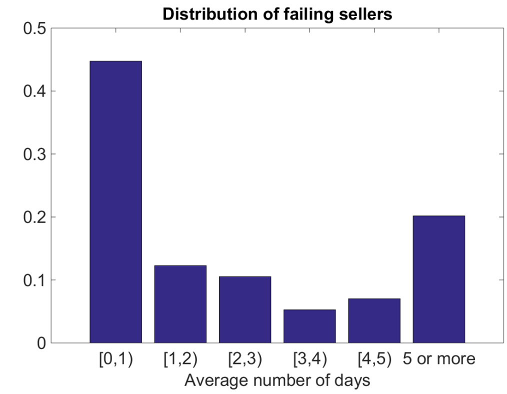
Clearly, these reasons are not mutually exclusive; for example, an operational problem may stop a dealer from trading, which in turn may cascade into a chain of fails and produce a shortage of liquidity on a specific instrument. And supply/demand imbalances are often the main incentive for strategic fails. In our case, by choosing to consider very



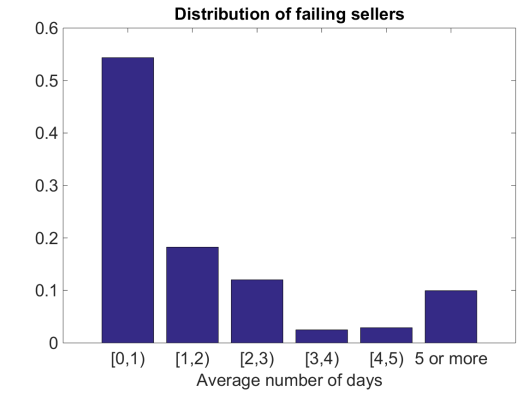
(a) Gilts



(b) FTSE 100



(c) Gilts



(d) FTSE 100

Figure 5: The graphs (a) and (b) show how volumes are distributed according to the business days between the intended settlement day and the day in which the trade was finally settled (or canceled). Graphs (c) and (d) show the distribution of failing sellers according to the average number of days their failed trades have remained outstanding before being settled or canceled. For each seller, this average is obtained by considering the average across ISINs and across all days in the period from October 3, 2016 to March 27, 2017.

liquid instruments we can assume that lack of supply is not a main driver of the fails observed.

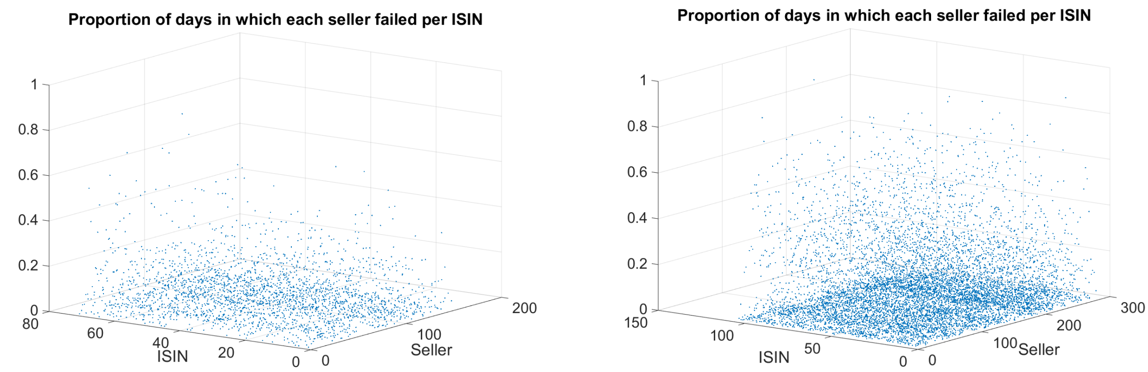
5.1 Clustering of fails

As a first step in the analysis, we explore whether fails cluster around particular ISINs and/or around specific participants. Clustering around particular ISINs would suggest that fails are linked to specific securities becoming special, at least temporarily. In principle, given that both gilts and FTSE 100 are liquid securities, we do not expect any significant clustering around ISINs. On the other hand, clustering of fails around specific participants could indicate that fails are linked to idiosyncratic factors, like operational limitations or to strategic behaviour.

To capture these effects we calculate how often each participant fails to deliver each ISIN during the sample period, expressing this as percentage of the total days in the sample. In other words, we consider the function

$f(X,i) =$ proportion of (business) days in which the participant X failed to deliver security i

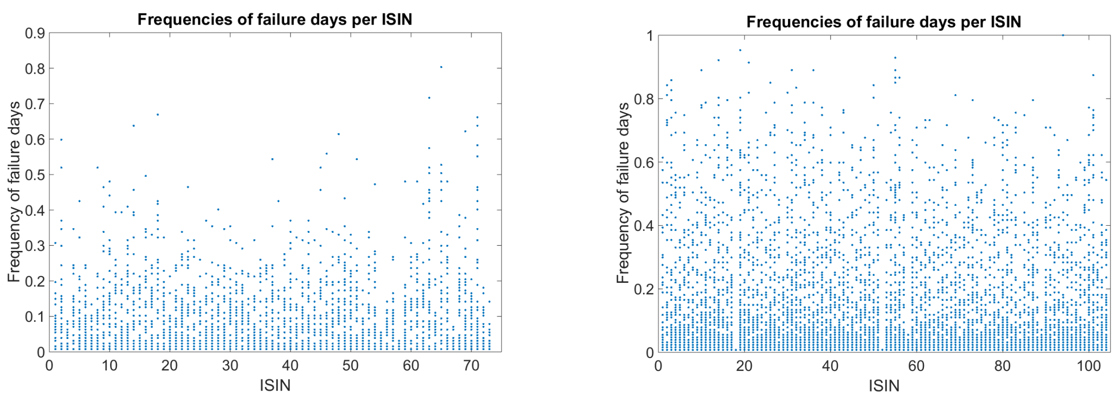
Figure 6 displays the values of f for gilts and FTSE 100.



(a) Gilts (b) FTSE 100

Figure 6: Proportion of days the participant X failed to deliver the security i on time during the sample period. Each dot indicates the proportion of business days in which seller X failed to deliver security defined by ISIN i.

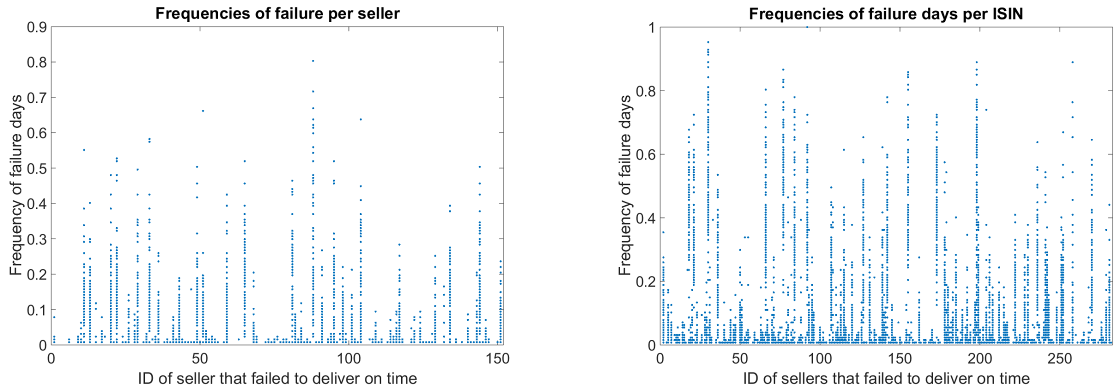
One way of visualizing the different behaviour across ISINs or participants is to consider the corresponding 2-dimensional projections of the plots in Figure 6. For example, Figure 7 displays the projection on the $f(X,i) - i$ plane: each dot corresponds to a seller and represents how often the seller failed to deliver the corresponding ISIN. A simple inspection suggests higher fail rates but more homogeneously distributed across sellers in the case of FTSE 100. In particular, in FTSE 100, almost all ISINs present relatively high frequencies of fail and they do so across a large number of sellers.



(a) Gilts (b) FTSE 100

Figure 7: How often each ISIN was failed to settle by a selling counterparty

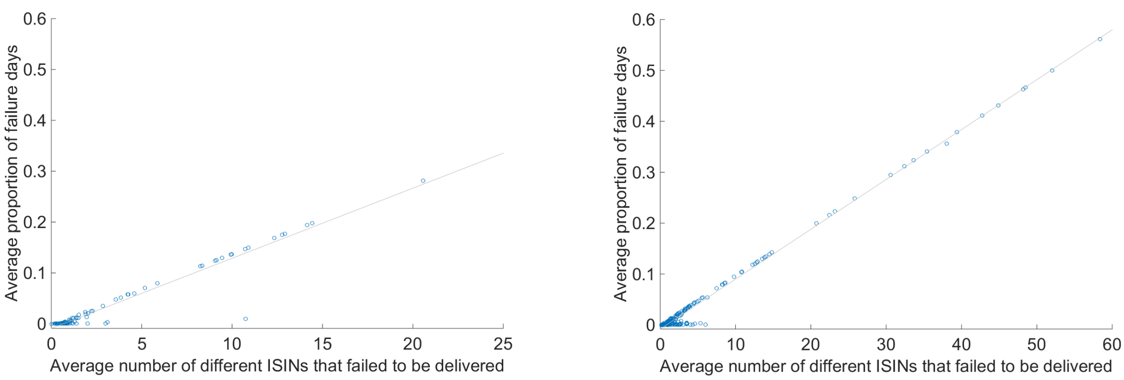
Figure 8 displays the projection on the $f(X,i) - X$ plane. It shows the fail pattern for each participant that failed at some point through the sample. In this case, for each participant that failed, each dot indicates the frequency of days failed on an individual ISIN.



(a) Gilts (b) FTSE 100

Figure 8: How often each failing market participant failed to deliver a security. For a given participant, each dot corresponds to an ISIN he failed to deliver.

We observe that, both for gilts and equities, some participants stand out from the rest by the high proportion of days in which they failed. Moreover, the plots suggest that sellers that often fail also tend to do so on many different ISINs, particularly in the case in FTSE 100. This suggests that more frequent fails are likely to be correlated with a larger number of failed ISINs. Indeed, if operational limitations are one of the main drivers of fails, one would expect a positive correlation between the diversity of the portfolio being settled by a participant and the frequency with which he fails to settle. To examine whether this could be the case, the scatter plots in Figure 9 show, for each participant, the relation between its frequency of fails and the number of different ISINs on which the participant has failed. In the case of FTSE 100 this relationship is almost perfectly linear, suggesting that participants selling a large variety of ISINs face more challenge in managing their settlement operations.



(a) Gilts (b) FTSE 100

Figure 9: The figures show the relation between the average percentage of daily fails for each participant, against the variety of ISINs the participant has failed to settle.

On the other hand, in the case of FTSE 100, the correlation between the ratio of fails and the total volume of fails is significant and negative (Figure 10). In other words, those sellers with higher fail ratio tend to be participants with low volume of trading, and vice versa. In the case of gilts, the relation is also negative but does not appear to be statistically significant.[9]

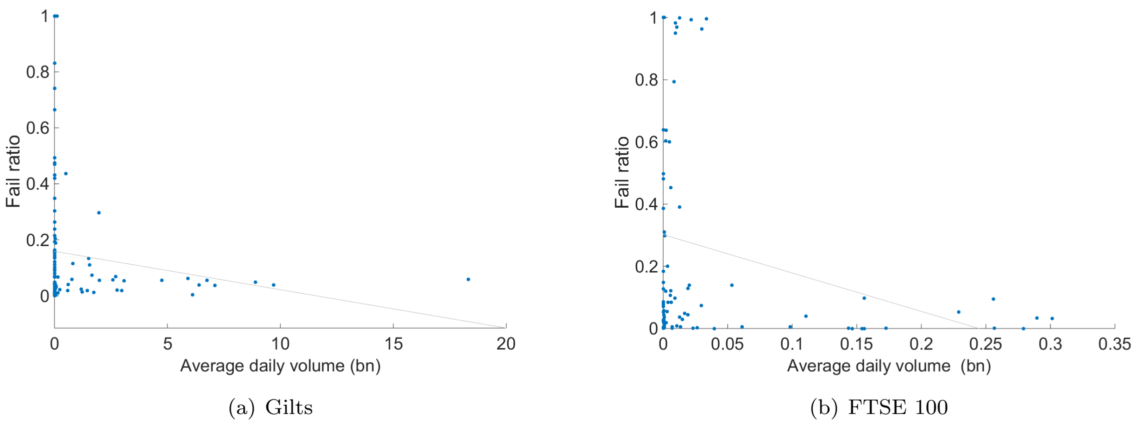


Figure 10: The figures show the relation between the total volume of trading for each failing seller, against the seller's fail ratio.

The above discussion points to fails being more linked to specific participants rather than to singular ISINs, which suggests that, in our sample, operational frictions and cascade effects play a more important role in settlement fails than supply/demand imbalances for specific ISINs. So we now turn our attention to cascade effects.

6 Cascade effects

In some cases, a participant A may fail to deliver a security simply because he was relying on receiving the same security from another participant B, who failed. In principle at least, participant A has the option to avoid failing by borrowing the security, but this may come at a cost. And in more interconnected markets, one would expect a higher degree of fail propagation. For example, in their study of the US markets, Iyer and Macchiavelli (2017) confirmed the importance of cascades of fails and the systemic nature of settlement fails by showing that at dealer level, failing to receive a security is passed 90% to 100% into failing to deliver the same security, and that this is true for a wide variety of asset classes.

In this section we will examine how cascades of fails contribute to the number of settlement fails observed on each day for each ISIN and each participant in the gilts and FTSE 100 markets. We will do this by, first, documenting the characteristics of the network of fails in a static setting (that is, analyze the network structure on a fixed random day) and then describing how fails evolve over time. We also develop an algorithm

[9]To avoid very low volumes distorting the results, we have also tested the relationship considering only volumes above 0.05 bn for Gilts or 0.01 bn for FTSE 100. The results remain similar.

to separate the fails due to cascade effects from the aggregate fails of each participant on each day for each ISIN.

6.1 Static network characteristics

To analyze the static security settlement network we randomly pick one day and we consider both successful and newly created failed transactions ($n_t(k) = 0$) on that day. Summary statistics of the one-day data set are showed in Table 2.

Table 2: Summary statistics of transactions on a randomly selected day.

	Gilts	FTSE 100
Number of transactions	10,988	67,562
Number of ISINs traded	72	98
Number of sellers	86	239
Number of buyers	89	243
Total volume of securities traded (bn)	131	4.95
Total value of securities traded (£bn)	175	21.23

We will consider participants as nodes in a directed network where a link from node A to node B reflects the obligation of A (the seller) to deliver a specific ISIN to B (the buyer). The links will be weighted by the volume of the obligation between pairs of nodes. To get a general picture of the basic characteristics of the network, including its connectedness and the expected number of ISINs traded among participants, Table 3 displays the descriptive statistics of sellers according to the ISINs they sold, and of the corresponding proportion of ISINs they failed to deliver on the intended settlement date. It turns out that the average FTSE 100 participant traded more ISINs than the average gilts participant (27 compared to 19), which indicates that FTSE 100 settlement network is more connected — in terms of number of links and number of nodes — than the gilts network, a result which seems confirmed when we visualize the networks, as in Figure 11. On the other hand, the average gilts participant failed on less of these securities. In both cases, the proportion of failed ISINs is similar (around 5%).

Table 3: Summary statistics of all sellers on one day.

	Gilts				FTSE 100			
	Min	Max	Mean	S.D.	Min	Max	Mean	S.D.
# of ISINs sold	1	72	18.76	22.065	1	98	26.9	31.60
# of ISINs failed	0	12	0.799	1.85	0	45	2.11	6.191
Proportion of ISINs failed	0	1	0.050	0.158	0	1	0.046	0.119
Proportion failed (in volume)	0	1	0.039	0.148	0	1	0.033	0.123

From a dual perspective, we also characterize the settlement networks for each ISIN. Table 4 presents some descriptive statistics of ISINs according to the number of sellers that traded them. The statistics show that, on average, more counterparties trade FTSE 100 than gilts (119 against 52) and that FTSE 100 has a higher ratio of fails. In addition, we can also distinguish the networks by measuring their average network density, which is defined as the ratio of the average number of directed edges to the maximal possible number of directed edges, i.e., the square of the number of nodes in the network. The settlement network for gilts has an average network density of about 0.0357, while the measure for FTSE 100 is about 0.0260. Even though the FTSE 100 market has more edges, because of the larger number of participants, its network density is actually lower than that of the gilts market.

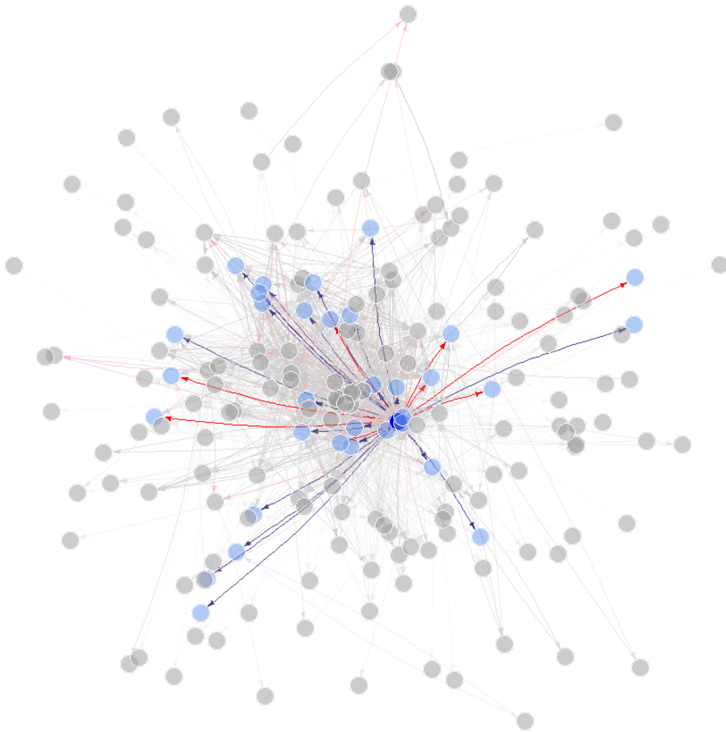
Table 4: Summary statistics of all ISINs traded in one day.

	Gilts				FTSE 100			
	Min	Max	Mean	S.D.	Min	Max	Mean	S.D.
# of nodes	23	76	51.70	10.6	68	210	118.81	35.88
# of links	24	233	95.54	36.35	168	865	366.54	165.67
# of fails	0	22	2.15	3.279	0	70	14.418	14
Ratio against total number of fails	0	0.127	0.023	0.029	0	0.240	0.041	0.039
Ratio against failed volume	0	0.117	0.013	0.023	0	0.433	0.048	0.072

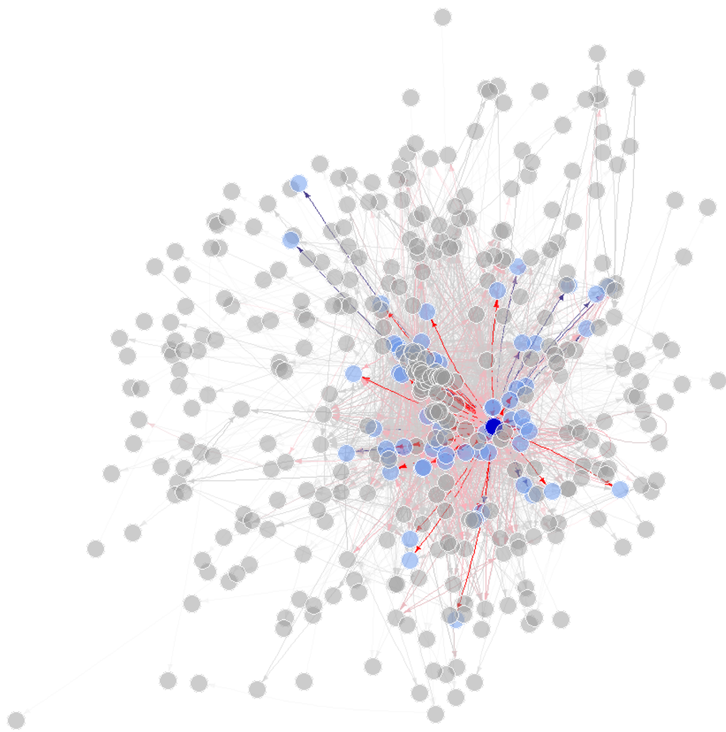
Figure 12 shows the settlement network structure of the ISINs with the largest contribution (in percentage) to the total number of fails in gilts and FTSE 100 (12.7% and 24% respectively) on the random day. Again, we observe that the settlement fail network for FTSE 100 is more connected and more complex than that of gilts and therefore potentially less stable, given the inverse relation between complexity and stability in networks (Bardoscia, M. et al., 2017). In the figure, the size of each node is proportional to its outdegree (that is, to the number of failed settlement agreements due from the participant).

Another aspect to examine is the relation between the nodes that fail the most variety of ISINs and those that are more central to the network.[10] The regression results in Table 5 show that, both for FTSE 100 and for gilts, the nodes that fail the most variety of ISINs are also the most central ones, with centrality measured by the extent to which a node lies on paths between other nodes (i.e. betweenness).

[10]We thank Guido Caldarelli for pointing out this aspect.

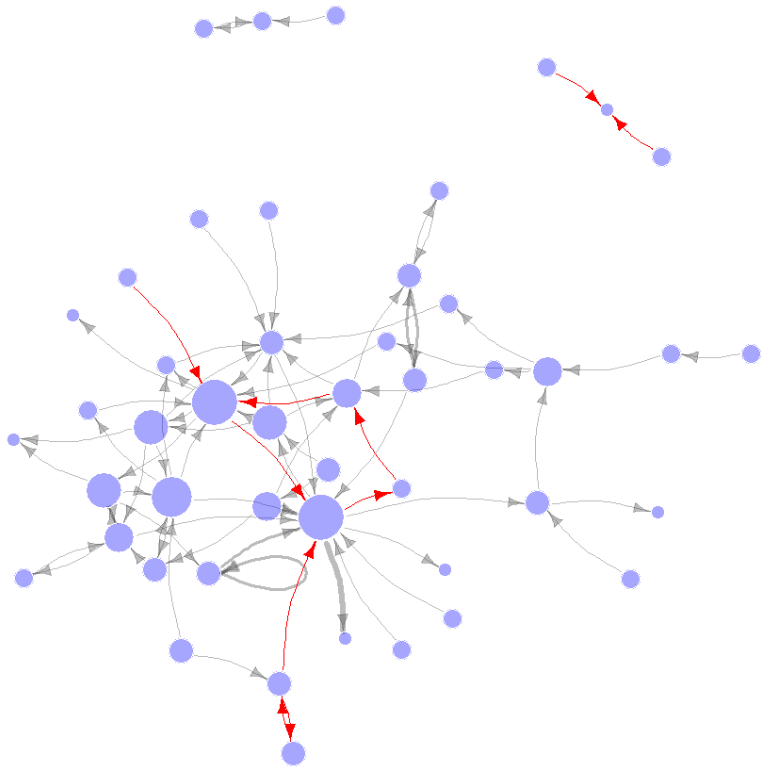


(a) Gilts

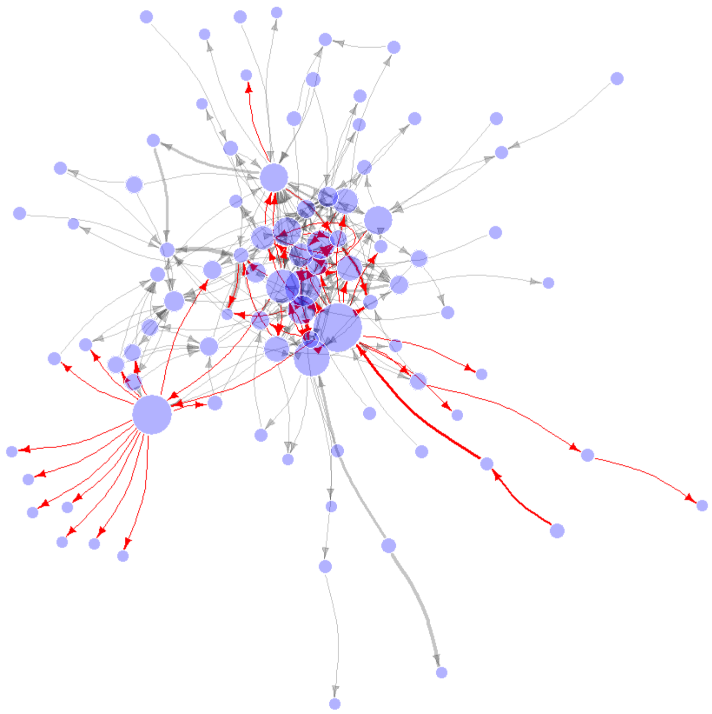


(b) FTSE 100

Figure 11: Security settlement networks of all ISINs on a random day. Participants are represented by circles (nodes). Gray linkages represent successful settlement, while pink linkages represent settlement fails. Note that there could be multiple ISINs traded between two counterparties, thus, there could be overlaps of linkages between certain nodes, shown as darker-color links in the network. For each security type, we highlight the seller that failed the most variety of ISINs with a dark blue node. Successful deliveries of securities are denoted as blue links, while failed deliveries are denoted as red links.



(a) ISIN with the largest contribution to total fails in gilts



(b) ISIN with the largest contribution to total fails in FTSE 100

Figure 12: These figures illustrate the settlement network of the ISINs with the largest percentage of number of fails in gilts and FTSE 100 respectively, on the random day. The node size is proportional to the participant's outdegree. Link widths are proportional to transaction volume of the contracts between counterparties. Gray links represent successful settlement, while red ones are fails. A loop from a node to itself is the transaction between different accounts held by the same participant.

Table 5: Results of regressing the number of fails of a node against its betweenness centrality.

	Gilts				FTSE 100			
	Estimate	SE	tstat	pvalue	Estimate	SE	tstat	pvalue
Intercept	0.4234	0.1496	2.8296	0.0052	1.5577	0.6904	2.2563	0.0247
Betweeness	60.1180	6.7253	8.9390	0.0000	603.9354	47.0021	12.8491	0.0000

6.2 Identifying cascade effects

For each participant, we define fails due to cascade effects as those fails which coincide with the participant not receiving the same ISIN from its debit counterparties. In this context, one could ask how often an ISIN is involved in a fail due to cascade effect (which relates to the connectedness of the settlement network for each layer of individual ISINs), or how often a participant fails in the presence of cascade of fails.

One important consideration when separating the fails due to cascade effects is that, because fails can last for several days, the cascade effects can propagate through time. For example, there may be trades which come due between two counterparties while their last trade is still unsettled for the same security. As a an illustration, in Appendix A we provide an empirical example of the dynamics of fails for one particular ISIN.

To separate the cascade effects from the new fails due from each participant on a specific ISIN, we proceed as follows: for each ISIN X and for each date t, let $\mathcal{S}_t(X)$ be the set of all participants that failed to deliver X on day t in a new transaction (that is, a fail with zero days outstanding). For new fails, the net obligation due from participant i is calculated as the quantity of new fails due from i on day t subtracting the new fails due to i on day t,

$$D_i^{new}(X,t) = D_{i,\bullet}^{\text{new}}(X,t) - D_{\bullet,i}^{\text{new}}(X,t). \tag{1}$$

On the other hand, we compute the net obligation due to i from previous days as follows: for each participant $i \in \mathcal{S}_t(X)$, we consider the difference

$$D_{-i}^{\text{old}}(X,t) = D_{\bullet,i}^{\text{old}}(X,t) - D_{i,\bullet}^{\text{old}}(X,t), \tag{2}$$

where $D_{\bullet,i}^{\text{old}}(X,t)$ is the total volume of fails due to i which have been one or more days outstanding at time t and $D_{i,\bullet}^{\text{old}}(X,t)$ is the total volume of old fails due from i. Both terms are non-negative.

To capture the fact that some new fails can be a consequence of past fails that are still outstanding, we define the *credit limit for network fails* for participant i on day t as the allowance for i to fail that is counted as network fails due to previous fails which are still unsettled on day t, that is

$$C_i(X,t) = \max(D_{-i}^{old}(X,t), 0). \tag{3}$$

Then, the net obligation due from participant i after separating the fails due to cascade effects is

$$D_i^{nc}(X,t) = max(D_i^{new}(X,t) - C_i(X,t), 0). \tag{4}$$

Subtracting the net obligation from the total gross new fails due from i gives us the new fails due to cascade effect by i on day t;

$$D_i^c(X,t) = D_i^{\text{new}}(X,t) - D_i^{nc}(X,t). \tag{5}$$

To measure the impact of cascade effects on new fails, we can estimate, for each seller i, the ratio of cascades of fails across ISINs relative to the total number of new fails:

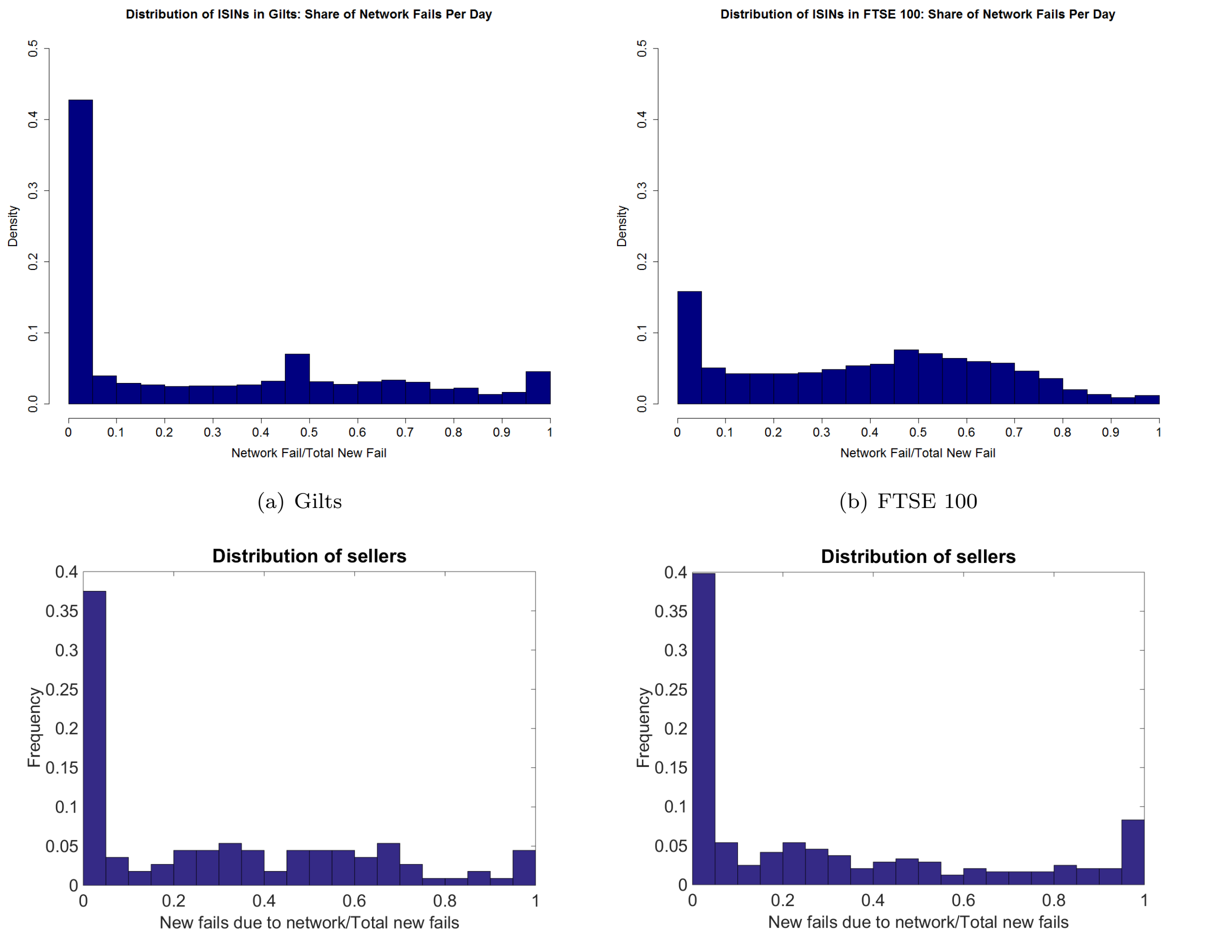
$$FR^c(i) = \frac{\sum_t \sum_{X\in\mathcal{X}} D_i^c(X,t)}{\sum_t \sum_{X\in\mathcal{X}} D_i^{new}(X,t)}. \tag{6}$$

where $\mathcal{X}$ denotes the set of ISINs. Similarly, if $\mathcal{S}$ denotes the set of failing sellers, we can consider, for each ISIN X, the ratio of cascades of fails across sellers

$$FR^c(X) = \frac{\sum_t \sum_{i\in\mathcal{I}} D_i^c(X,t)}{\sum_t \sum_{i\in\mathcal{I}} D_i^{new}(X,t)}. \tag{7}$$

Panels (a) and (b) in Figure 13 show how ISINs are distributed according to the ratio $FR^c(X)$. We observe that for more than 40% of all the gilt ISINs failed (as indicated by the first column in Panel (a)), they are not significantly due to cascades of fails. In other words, the cascade effect accounts for less than 5% of their total new fails. However, this proportion (as indicated by the first column in Panel (b)) drops to only 15% in the case of equities. This suggests that, while both in gilts and equities cascade effects play a role, the effect for ISINs in FTSE 100 is stronger. These differences could be driven by the different market structures and the different type of participants.

When we turn to the ratio of cascade effects per seller, $FR^c(i)$, Panels (c) and (d) in Figure 13 show that in both markets, about 40% of all the sellers are not significantly affected by cascades of fails. These could be the financial institutions that have some operational problems but are not at the core of the settlement fail network. Whether their fails contribute or not to cascades of fails in the settlement system will depend on their out-degrees in the settlement network. On the other hand, the remaining 60% of the sellers is affected to different degrees by cascade effects. In particular, the thick right tails of the distributions demonstrate that around 5% to 10% of all the participants suffers from cascade effect for almost 100% of their settlement contracts. These could be institutions with a large in-degree or betweenness centralities where, because of their special position in the network, fails are mostly caused by their counterparties failing rather than by individual operational problems or decisions.



(a) Gilts

(b) FTSE 100

(c) Gilts

(d) FTSE 100

Figure 13: The distribution of ISINs and sellers according the proportion of network fails (cascade effects) out of total new fails. Panels (a) and (b) show the distribution per ISIN. Panels (c) and (d) show the distribution per seller.

Among the institutions with a large amount of cascade effects we should single out the case of central counterparties (CCPs). Since CCPs stand on the middle of the trade to guarantee the performance of the counterparties then, they are - almost by design - fully impacted by cascade effects. In fact, because of their role, CCPs add a certain amount of double counting of fails. A trade that participant A failed to settle with a CCP, which in turns fails to counterparty B will appear as a chain of two fails, although it is originally one single transaction, the original sale from A to B, that has failed. In our sample we have five participants which are CCPs and these are amongst the firms with ratios close to 1 in Figure 13.

7 Buy-in strategies

In this section, we depart from the specifics of a particular market and discuss, from a theoretical perspective, a buy-in strategy to resolve a chain of fails which maximizes settlement with minimal cost.

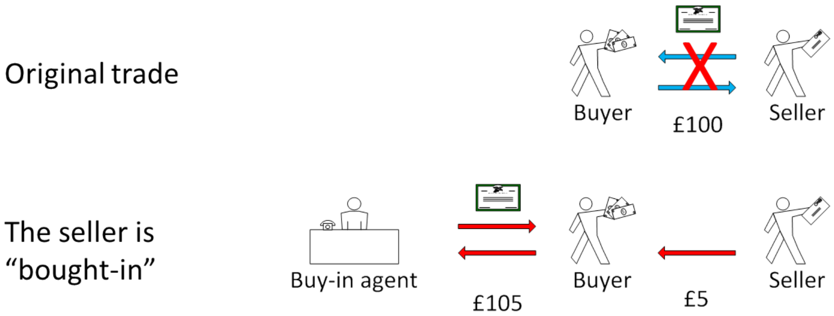


Figure 14: An illustration of the buy-in mechanism. After the seller fails to deliver the securities, the buyer can instruct a third party to source the security. Any difference in the market price is paid by the seller.

7.1 The buy-in mechanism

As noted above, buying-in is a mechanism that is used as a remedy that the buyer of a security can exercise in the event that the security fails to be delivered. Although the implementation details varies across settlement systems and jurisdictions, the principle behind it is to restore the economic positions the counterparties would have been in had the original transaction settled. And it usually takes the form of a contractual right by which, in the event that the seller fails to deliver the securities on the agreed settlement date, the buyer can enforce delivery of the securities to replace the original transaction. This could be done, for example, by instructing a third party to purchase and deliver the securities, or by conducting an auction. Any losses derived from differences between the price of the original transaction and the buy-in price are paid by the failing counterparty. An illustration of a typical buy-in process is displayed in Figure 14.

While the purpose and benefits of a buy-in process are clear, in practice a buy-in process may be constrained by operational frictions (Hill, 2015) and by liquidity conditions in the market. Because of these constraints, it would be desirable to identify buy-in strategies that could have maximal impact in resolving a settlement fail chain at a minimal cost. However, because of the potential complexity of a network of fails, designing an efficient buy-in strategy may not be straightforward. For example, when a network of fails has resulted in closed chains of fails it may be difficult to determine which participants need to be bought in and by which amounts, in a way that the outcome is consistent across all the nodes in the network and the curing effect of the buy-in is maximized.

As an example, assume that, at any time t_0, we can observe the settlement fail network for each ISIN traded, including counterparties and the volume of the securities that failed to deliver. The participants (that is, the nodes in the network of an ISIN) can be divided into three categories:

A) Participants who are only buyers of securities, therefore do not need to be bought-in.

B) Participants who are pure sellers and therefore must be bought-in for sure.

C) Participants who are both buyers and sellers. Whether they need to be bought-in and by how much will depend on how much they will receive from other nodes once the buy-in process is initiated and how much they have to deliver to other nodes.

To illustrate this, Figure 15 shows the three types of participants in the settlement fail network of one ISIN in gilts. The circle nodes are the pure buyers of the ISIN; the square nodes are the pure sellers of the ISIN; and the triangle nodes are the ones who both buy and sell the ISIN. We would like to find the buy-in strategy that solves the network of fails with the minimal amount of securities injected to the settlement network.

7.2 Optimal buy-in strategies

Since a network of settlement fails is similar to that of a network of payment obligations, to identify a way to complete the buy-in process with the minimum participants being bought-in, we adapt the framework developed by Eisenberg and Noe (2001) (E-N hereafter) by interpreting their external operating cash-flows as securities that are borrowed or bought from the market. One important difference is that, while E-N focuses on the network of payment liabilities that will mature at some future date, in our case we are looking at the nominal settlement obligations that remain when settlement has already failed to happen. In other words, we focus on the time when a buy-in process will be defined.

First, for any given ISIN X, we represent the network structure of fails at time t_0 by an $n \times n$ liabilities matrix $D = D(X)$, where n is the number of participants involved in the network and the matrix element D_{ij} is the volume due from participant i to j. In other words, i needs to deliver D_{ij} units of securities to j. As usual, we assume $D_{ij} \geq 0$ for $i \neq j$ (all claims are non-negative), and $D_{ii} = 0$ (no node has claims against itself). The sum $D_i = \sum_{j=1}^{n} D_{ij}(X)$ represents the total obligation of i to all other nodes in the network, and $D = (D_1, ..., D_n)$ is the associated vector (*total obligations* vector) for all the participants.

We define a *buy-in vector* as a vector $b = (b_1, ..., b_n)$, where $b_i \geq 0$ represents the number of securities that will be bought-in against node i; that is, the amount of securities that participant i needs to borrow or buy to fulfill some or all of the obligations that it has towards other nodes. These securities are exogenous to the network and we can interpret them as being lent by the market (which could also be represented as an external node with edges pointed to the nodes in the network).

Assume that at time $t_1 > t_0$, once the buy-in b has been defined,

1. all participants will pass on the securities they receive according to their obligations. In particular, at this point there is no strategic behaviour or operational frictions that impedes delivery; and

2. the securities will be allocated proportionally based on obligations to their credit counterparties. In other words, each debit counterparty i will pass on the securities

to its credit counterparty j based on the below proportion:

$$\pi_{ij} = \begin{cases} \dfrac{D_{ij}}{D_i} & \text{when } D_i > 0, \\ 0 & \text{otherwise.} \end{cases}$$

Let p_i be the total number of securities delivered from node i to all other nodes at time t_1 and let $p = (p_1, p_2, ..., p_n)$ be the vector of total securities delivered. Then, the total volumes of the ISIN X received by i are

$$\sum_{j=1}^{n} \pi_{ij} p_j. \tag{8}$$

A *clearing vector* for the system (D, b) is a vector $p^* \in [0, D]$ satisfying that

1. nodes cannot deliver more than what they actually have, and
2. obligations are either settled in full or all securities available to the node are delivered to creditors.

This implies that p^* is a clearing vector if and only if it satisfies the following set of non-linear equations:

$$p_i^* = \min\{D_i, \sum_{j=1}^{n} \pi_{ij} p_j + b_i\}, \quad \text{for } \forall i = 1, ..., n. \tag{9}$$

A direct application of E-N framework in this context implies that, under mild regularity conditions, given a system (D, b), a settlement vector $p = p(b)$ exists and it is unique.[11] The question we raise is how to characterize the minimal buy-in vector $\bar{b}$ that guarantees full payments and, when the full buy-in is not possible, how to proceed to achieve the most efficient settlement at the lowest cost.

Proposition 1. *In a static security settlement fails network assume that, after a buy-in process is defined, all participants will pass on the security they receive according to their obligations, and all counterparties have the same priority to receive the security. Then, for any security X,*

a) The minimal buy-in vector needed to fulfill all settlement obligations is $\bar{b} = \max(D_i - \sum_{j=1}^{n} D_{ji}, 0)$;

b) Given any buy-in vector b', there is a buy-in vector b, such that $p(b) = p(b')$ and for which the minimum set of participants that need to be externally bought-in is composed of the participants with strictly positive net obligations. Moreover, $\sum_i b_i \leq \sum_i b_i'$.

Proof. Assume that, at time t_0, we observe the security settlement fails network for any given ISIN X. By definition, if all settlement obligations in the system are met in full then, $p^* = D$ which, according to equation 9, implies that $b_i \geq D_i - \sum_{j=1}^{n} \pi_{ij} p_j$ (node

[11] In what follows, we consider the pointwise ordering of vectors; that is, for any two vectors x and y in $\mathbb{R}^n$,

$$x \leq y \iff x_i \leq y_i, \qquad \text{for all } 1 \leq i \leq n.$$

i has enough securities to cover its obligations to all j and $\sum_{j=1}^{n} \pi_{ij} p_j = \sum_{j=1}^{i} D_{ji}$ (all obligations to i are paid in full), for all i. This implies that the minimal buy-in vector is defined as

$$\bar{b}_i = \begin{cases} 0 & \text{if } \sum_{j=1}^{n} D_{ji} \geq D_i, \\ D_i - \sum_{j=1}^{n} D_{ji} & \text{otherwise.} \end{cases}$$

In particular, the external buy-in needed for each participant is the netted obligation it owes to other counterparties when the netted quantity is strictly positive, which proves the first point.

To prove (b) we define a buy in process b as follows:

$$b_i = \begin{cases} 0 & \text{if } \bar{b}_i = 0, \\ \min(b'_i, \bar{b}_i) & \text{otherwise.} \end{cases} \tag{10}$$

Clearly, $0 < b \leq \bar{b}$. On the other hand, if p' is the clearing vector associated to b', we need to show that it is also the clearing vector associated to b. To see that this is the case, we observe that, if $b'_i < \bar{b}_i$ then, by equation 10, we have that $p_i = p'_i$. On the other hand, if $b'_i \geq \bar{b}_i$, then equation 9 implies that $D_i \leq \sum_{j=1}^{n} \pi_{ij} p_j + b'_i$ and therefore $p_i = D_i = p'_i$. □

In summary, the result shows that an optimal buy-in strategy always exists where only the nodes with positive net obligation positions are bought in. And that this holds even if there are constraints in the amount of securities to be bought or borrowed and the optimal buy-in vector $\bar{b}$ cannot be achieved. It is worth noting that by assuming that all participants will pass on the security they receive according to their obligations, we are effectively assuming that, once the buy-in is initiated, there is no strategic behaviour.

The application of the above proposition is illustrated in the example shown in Figure 15. It displays the settlement fail network for a random gilt ISIN on a random day in our sample data. All the orange nodes consist the minimum group of participants need to be bought-in. Blue nodes are the ones receiving or passing on the buy-in security. Numbers on the nodes represent participant identity, and numbers on the directed edges represent the obligation volume (in 10,000s) between two counterparties. Edges point from the seller of the security to the buyer. All obligations in the network will be fulfilled after the orange nodes being bought-in and securities being passed on. Given that security pass-ons are proportional to participant obligations, the proposed buy-in amount will fulfill all obligations in the network, therefore, there is no incentive for any participant to hold extra security rather than pass it on to its credit counterparties.[12]

[12] Assume that there is no change of security demand of each participant during the buy-in process.

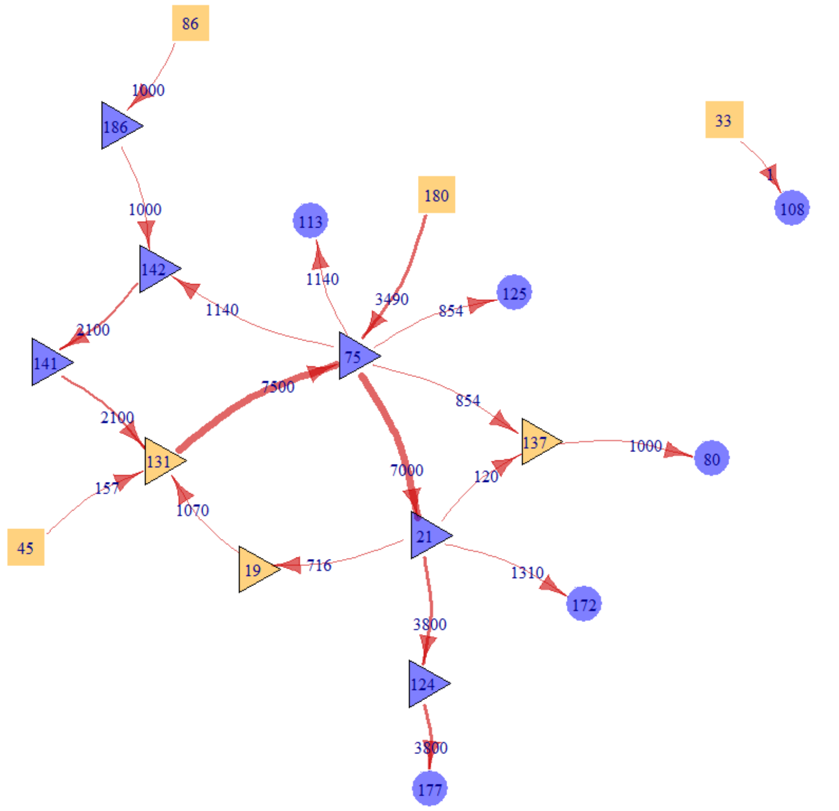


Figure 15: Example of an optimal buy-in solution of a random gilt ISIN. Numbers on the nodes represent participant identity. Numbers on the directed edges represent the obligation volume (in 10,000s) between two counterparties. The width of edges are proportional to obligation volumes. The figure distinguishes the three types of participants in the settlement fails network: (1) circle nodes are the pure buyers of the ISIN; (2) the square nodes are the pure sellers of the ISIN, therefore, need to be bought-in; and (3) the triangle nodes are the ones who both buy and sell the security.

Notice that in this example, participants 75, 21, 19, and 131 formed circular connections. Among them, participants 75 and 21 do not need to be externally bought-in, as they will received enough security from their debit counterparties to deliver it to their credit counterparties. Nevertheless, participant 19 receives 7,160,000 units of the security from participant 21, which is insufficient to fulfill its obligation to participant 131. Therefore, participant 19 needs to be externally bought-in for $10,700,000 - 7,160,000 = 3,540,000$ units. Similar rule applies to participant 131. Based on Proposition 1, we do not need to decide at which node we should start the buy-in process in the circular connections. We can directly identify that participants 131 and 19 are the ones with positive net obligation positions, and the volumes of external buy-in needed equal to their net obligation volumes respectively.

One of the assumptions of the model is that securities will be passed to the buyers on a pro-rata basis. In reality, settlement systems may have different rules to prioritize settlement. For example, the settlement system may allocate securities giving preference to older trades, or instead allocate them so that the maximum number of trades is settled. The results of Elsinger (2011) show that we can relax the condition of equal priority in

Proposition 1 to capture these more general situations where securities are settled using some prioritization mechanism.

8 Summary and concluding remarks

In the first part of this paper we study the network characteristics of settlement fails in the UK equities and gilt markets. For this purpose, we first document some stylized facts about settlement fails in each of these markets, including their persistence across time.

We next show that fails cluster around specific sellers and that there is a strong correlation between the frequency of failing and the diversity of the portfolio on which the failing participant trades. Furthermore, sellers' fail behaviours are significantly correlated with their positions in the network. In contrast, fails appear to be homogeneous across ISINs. Altogether, these results suggest that settlement fails in these markets are mostly driven by operational problems or by the impact of cascades of fails, rather than by factors linked to individual securities.

A fail to settle a trade not only may impact the ability of other participants to meet their obligations (as in a chain of fails) but, as they persist through time, fails may also have an impact on new trades that are due on subsequent days. To analyze these dynamic features we distinguish between new and old fails and we also differentiate those fails that are due to cascade effects (either on the day or from previous days) from those that are not a consequence of cascades of fails. We find that only a small proportion of the fails in FTSE 100 securities (less than 17%) was not part of a cascade of fails. By contrast, for gilts more than 40% of ISINs that failed was not significantly involved in a cascade of fails. This suggests that, while both in gilts and equities cascade effects play a role, the effect is stronger in the FTSE 100 market. When we turn to the ratio of cascade effects per seller, the analysis shows that, in both markets, only around 40% of all the sellers are not significantly affected by cascades of fails. These results stress the importance of resolving or mitigating cascades of fails for improving settlement efficiency.

In line with the above, in the second part of the paper we focus on the "buy-in" process (a process used to mitigate the impact of a fail) with the aim of identifying the optimal buy-in strategy to resolve a cascade of fails. For this purpose, we adapt the framework developed by Eisenberg and Noe (2001) to the case of a network of fails, interpreting the buy-in of securities as the exogenous cash-flows. Assuming that participants will pass the securities they receive, we show that, for any buy-in strategy, there is always a buy-in strategy which achieves the same level of settlement efficiency and where the minimal set of participants needed to be bought-in is composed of the participants with strictly positive netted obligations.

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Appendix A An example of cascade effects dynamics

In this section, we present an empirical example to illustrate how networks of fails can evolve through time. The example also demonstrates that a fail from previous days could be one of the causes of new fails on later dates. In other words, the cascade effects can have a time lag effect.

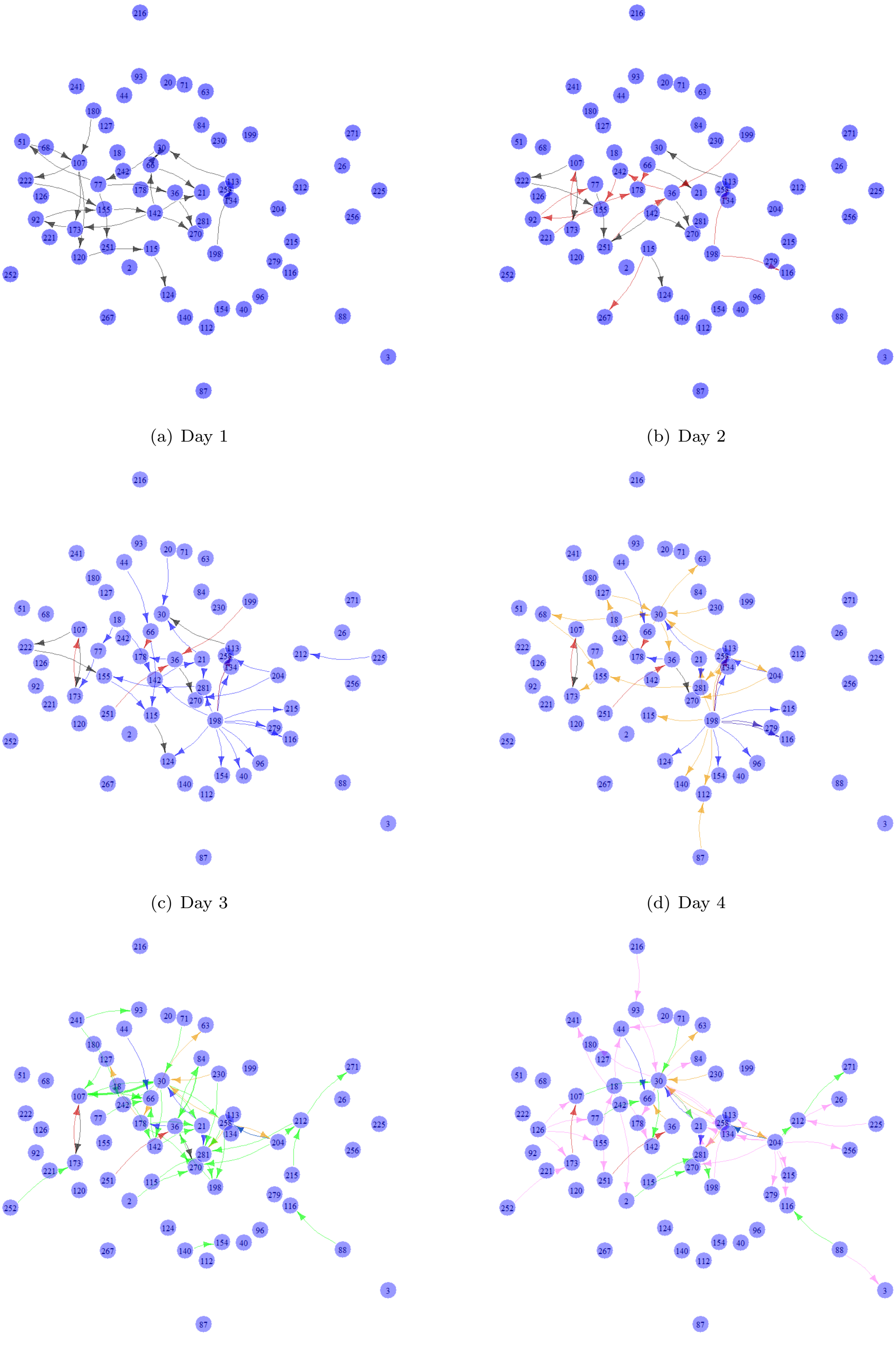
Figure A-1 displays the dynamics of a network of fails for a randomly selected ISIN over a random time interval of 6 business days in our sample. Let $t = 1, 2, ..., 6$ in the example. In the figure, the numbers on the nodes represent participant identity. We include all the participants that had failed to deliver the security on time for at least once during the period.

In Panel (a) of the figure, new settlement fails on day 1 ($n_1(k) = 0$) are presented in black linkages. On day 2, as displayed in Panel (b), new fails ($n_2(k) = 0$) are presented in red edges. There were 10 obligations that remained unsettled from day 1, some of which caused fails on day 2. For example, participant 251 had been waiting for security deliveries from participants 36 and 77 since day 1, and it failed to deliver the security on day 2 to participant 36.

On day 3 in Panel (c), new settlement fails ($n_3(k) = 0$) are in blue linkages. Participant 36 failed to deliver the security to participant 178, because it had been waiting for the deliveries from participants 251 and 199 since day 2.

New settlement fails on day 4 ($n_4(k) = 0$) are represented in orange linkages, among which some fails were caused by fails from day 3 and day 2. For instance, participant 178 had been waiting for security deliveries from participant 36 since day 3 and from participant 66 since day 2, so it defaulted to participant 127 on day 4. This fail continued on day 5, causing participant 127 to fail to deliver the security to participant 107 on day 5, which is represented in green edges in Panel (e). Note that the contracts between participants 107 and 173 from the initial two days still remained unsettled on day 5.

On day 6 in Panel (f), new settlement fails ($n_6(k) = 0$) are in pink edges. Again, some of the fails were due to the remaining fails from previous days. For example, obligations among participants 30, 21, 258, and 204 remained unsettled since day 3, 4, and 5, causing more fails with other counterparties on day 6.



(a) Day 1 (b) Day 2

(c) Day 3 (d) Day 4

(e) Day 5 (f) Day 6

Figure A-1: An empirical example of the dynamics of a network of fails of one random ISIN